Aharonov-Bohm and Aharonov-Casher Effects: Connections to Dynamics of Topological Singularities

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We analyze the physical processes involved in the Aharonov-Bohm (A-B) and the Aharonov-Casher (A-C) effects, showing that an incomplete A-B effect knowledge can lead a totally wrong conclusion on the A-C effect. Based on this we demonstrate that the Magnus force, the net force, is the only transverse force on a moving vortex, in analogous to the net charge in A-C effect. This conclusion has been arrived both theoretically and experimentally.

Let us begin with a well accepted situation, the connection between the A-B effect [1] and the A-C effect [2]. The A-C phenomenon is the effect of charges on a moving magnetic flux line, the dual effect of A-B phenomenon of the magnetic flux line effect on moving charges. Within the non-relativistic formulation, those two effects can be formally related to each other by a Galilean transformation. However, the crucial difference between A-B and A-C effects in real calculations is that, for the A-B effect a moving charge 'sees' the net magnetic flux, but for the A-C effect a moving magnetic flux line 'sees' the net charge. It is rather easy to conceive the situation of a flux line in a condensed matter: suppose one knows perfectly well a large A-B effect for conduction electrons, can he draw a definite conclusion on the A-C effect for the moving flux line? The answer is NO, because the net charge for the A-C effect comes from various contributions, and the contribution from conduction electrons is only one of them. For example, the A-C effect can be zero in the case of charge neutrality, and, be either positive or negative depending on the net charge feel by the flux line. Until all charges, valence electrons, ionic background, quarks, ..., have been considered, the A-B effect for conduction electrons alone can not yield any information on the A-C effect for a moving flux line. This illustrates the pitfall in using the incomplete A-B effect information to

deduce the A-C effect. The best way to find the effect of net charges on a moving magnetic flux line is a direct calculation following this flux line, as already suggested by Aharonov and Casher.

Now we are ready to discuss the transverse force on a moving vortex. The phonons act like, which may sounds natural to some authors, conduction electrons, and a vortex like a magnetic flux line. Let's accept this analogy without further question, though it may not be true. The phonons scattering off a vortex resembles the A-B effect, while a moving vortex picks a possible transverse effect from phonons resembles the A-C effect. According to above analysis of the connection between A-B and A-C effects, the transverse effect due to phonons cannot tell us anything about the net transverse force felt by a moving vortex. One has to consider the normal fluid, superfluid, the total fluid, and other contributions involved. This phonon effect can be much larger than the net transverse force on a vortex, and can even carry a negative sign. In analogous to the calculation of A-C effect, the exact result, obtained by calculation the Berry phase [3], or, explicit counting individual state contributions [4], shows the magnitude of the net transverse force on a moving vortex is determined by the superfluid density. The absence of any additional transverse force has also been shown in a recent path integral derivation of vortex dynamics. [5]

We should emphasize that there are many possible individual contributions to the transverse force. They do not all carry the same sign. The topology of the vortex requires that their sum, the net effect, is universal, determined only by the superfluid, although any one of such contributions may have a magnitude larger than the net effect. Simultaneously, there are equal number or more contributions to the longitudinal force, the friction. The important fact is that they all carry the same sign. Hence one has to sum all of them up to determine the friction. This is a detail-sensitive procedure which shows friction has no universal value. This striking difference between the transverse force and friction may have caused a considerable amount of confusion in the literature to understand the exact result on the transverse force.

It is also worthwhile to emphasize that, in addition to above theoretical analysis, well

accepted experiments, such as the vibrating wire [6], the vortex procession [7], as well as a recent vibrating reed experiment done in Umeå [8], have shown the non-existence of extra transverse forces.

Finally, we point out that the transverse force in the A-B effect has been well established [9], and routinely used in the study of quantum Hall effect [**?**] and vortex dynamics in Josephson junction arrays [11].

The conclusion is, the analogy to the Aharonov-Bohm and Aharonov-Casher effects has put the recent topological demonstration of the vortex dynamics on a firmer ground. There is no extra transverse force other than the Magnus force, shown both theoretically and experimentally.

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